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Structural approach to the inverse problems of computational diagnostics in cardiology

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Abstract
The methods to identify the structural invariants of the evolving test object using projection data are offered within the framework of the group-theoretical statistical approach. The essence of the inverse reconstructive problem within the structural approach is investigated. The features of the inverse problems arising in the study of developing open system are discussed. The advantages of the stroboscopic data acquisition in hardware–software implementation of the group-theoretical statistical reconstructive methods are explored. The concept of phase time is introduced, and the methods of stroboscopic reconstructive diagnostics in phase time are developed. The electrocardiogram (ECG)-controlled systems for X-ray medical reconstructive diagnostics are designed. The basic types of reconstructive — problems for evolving test objects are considered.

Keywords: Inverse problem; image reconstruction; structural invariants; stroboscopic methods; Lie group; statistical hypothesis; open system.

Introduction

Among the objects of nature and society a man has to deal within research and practice, “evolving” (or “dynamic”) objects are dominant [1,2]. Sufficiently, complex objects are the open systems [1] exchanging energy, information, and matter with the environment. It is obvious that organisms and their physiological subsystems are examples of open systems. High degree of generality of principles of the system approach (in particular, the principles of structural-system research [3]) allows us to use them both to study a variety of open systems and to manage them.

The subject of our special interest in this article is the cardiovascular system of a human body investigated by means of structural-system methods. A structurally oriented approach developed in Refs [4-7] is suggested to solve the problems of computational medical diagnostics in cardiology. It has been successfully applied to solve a number of crucial problems in nondestructive testing (NDT), in particular, the statistical group-theoretical methods for solving inverse reconstruction problems [4-6] are developed on its basis.

Since the object is complex, the open system is “multilevel” [1], and any description of the system is directed and “selective”.

The most important characteristic of the open system in its interaction with the environment is maintaining its integrity [1,2], therefore, maintaining certain structural invariants [3], which enables to distinguish it from the environment and consider it as a quasi-independent “piece of reality”. Changing, it remains unchanged, more precisely, changes occur mainly at the level of elements [3], whereas “structural core” is invariable. An open system (as well as any system) is characterized by its principled irreducibility to the sum of its constituent elements and it is impossible to derive features of the system (i.e., of “whole”) from features of these elements.

This paper assumes “relative stability” of the system and it does not consider the catastrophic ways of its development with the following destruction of the “structural core”. Nevertheless, the goals and objectives of the diagnostics require recognition of “pre-catastrophic” conditions caused by different factors leading to a “shift” of certain structural invariants.
Diachrony and Synchrony in the Study of Evolutionary Processes

Two approaches are applied to study the developing systems: a “diachronic” approach and a “synchronic” approach [8,9]. Within the diachronic approach, we consider the evolutionary sequence of the development of phenomena in time, whereas within the synchrony approach, we deal with their coexistence at a particular moment in time. This contrasting arose in the late 19th century after the structural linguistics was created by de Saussure [8,9], and the “synchronic” approach to language was developed as opposed to an evolutionary approach within comparative-historical linguistics. Later, these results to a great extent influenced the formation and development of structural methods in science.

Since the “synchronic” approach aims to study the structural and systemic-invariant properties of the objects, the synchrony is often interpreted as “timelessness”. In mathematics, the concept of “structure” is applied to the sets, in which the nature of the elements is not defined. This is closely linked to the prevalence of relational models in physical and mathematical sciences (and in systems analysis as well) stepping back from the “natural” properties of objects (in particular, from their “element base”) and concentrated on the study of structural and functional relations and relationship between the elements in the object itself, and between the objects.

In the structural approach, the focus of the research is shifted from the elemental (typically material basis of the object to its formalized transformations. In this case, the object under study is presented as a two-level system separating the invariant structure and the substratum) i.e., “sublayer”, the lower level at which the changes in the system basically occur. The “synchronic approach” is “almost synonymous” with the structural approach; however, it is used to emphasize the evolutionary aspects in the test object. The substratum for this object is a “phenomena level” or a “range of phenomena” associated with this object. The phenomena are “ephemeral” and unstable, whereas the structural core of the object is “relatively stable”.

The diachronic and synchronic (in other words, evolutionary and structural) descriptions of the object are mutually complementary, and as such, they do not contradict each other. However, structural description, which is usually associated with accuracy and objectivity, remains a conventional and preferred description in science. This is quite evident, since knowledge is aimed primarily at “constant”, at establishing laws and identification of invariants. The diachronic approach is concerned with the study of relations linking the elements that follow each other in time in the “phenomena stream”. They are not perceived as a whole by one and the same consciousness and do not form a system. The synchronic approach treats the relationships linking the coexisting elements. They are perceived as a whole by one and the same consciousness and form a system. We can say that the synchronic approach works directly with the “entire list” of the phenomena probable within this theoretical model.

In physical and mathematical sciences, the “synchronic” approach is well manifested in “conservation laws” and “integrals of motion” of the physical system, as well as in identification of its structural invariants under various transformations described in the language of the group theory [3].

We are developing a basically structural “group-theoretical statistical approach” to the study and diagnostics of the “evolving” objects, particularly related to diagnostic cardiology problems. The focus of the approach is primarily on identification of spatio-temporal invariants in evolutionary processes under the assumption that these processes are dynamically stable. The emphasis is on identifying invariants practically significant for diagnostics of various diseases.

Nevertheless, evolutionary (diachronic) approach within these studies cannot be “thrown overboard”. First, this is because diagnostically valuable invariant characteristics of the body should be reconstructed from the phenomena by solving an inverse problem. Second, the diachronic approach is relevant to study the impact of various external factors on the cardiovascular system, as well as “internal” factors appearing in the system in the process of evolution. This raises the problem of causality analysis based on the principle of dissymmetrization by Curie [5] within the group-theoretical statistical methods which is alien to the synchronic approach [4].

Inverse Reconstruction Problems for Open Systems

In the context of the strict structural approach, the theoretical model is considered to be both
the object to be reconstructed and a new structural whole.

Theoretical models developed to describe the behavior of objects in the process of evolution are “dynamic”. In other words, for the description of objects in them such concepts as “time”, “interaction”, “cause and effect”, etc., are used. Note that this is the lexicon mainly of a diachronic approach. As for theoretical model itself (i.e., considered as “whole”), it is the prerogative of the synchronic approach. Phenomena evolve, whereas the theoretical model and the structure of laws which governing these phenomena remain to be “timeless” and “have no cause”.

Thus, the inverse problem (as creation of modified model) implies a transition from the diachronic to the synchronic description of the system, whereas the direct problem considers the transition from the synchronic to diachronic description (i.e., from laws to phenomena).

The inverse problem is solved in the framework of the subject–object unity. The solution requires hypothetical subjective assumptions \( A_\alpha \) and verification of their consistency with the objective experimental data \( AO \) on the test object. According to Leibniz, \( A_\alpha \) is “truths of fact” ("vérité de fait") and \( A_\nu \) is “truths of reason” ("vérité de raison"). The principle of the integrity is essential for solution of the inverse problem. Assumption \( A_\alpha \) is an “integrating idea” (the organizing whole, which is brought into the solution by the subject); without which the problem remains “unsolvable” even if the entirety of the original data is obtained. In other words, the “integrating idea” is essential not only for solving of “ill-posed” inverse problems, but for all, without exception, inverse problems (Being core of new whole the assumption \( A_\alpha \) cannot be reduced to “a priory data”). Within the group-theoretical statistical approach, \( A_\alpha \) is group-theoretical hypothesis about the test object to be tested by statistical methods.

**Multivariance and Directedness in the Solution of Inverse Problems**

Since the object to be studied is multilevel, its information image may contain a variety of the “senses” (or “meanings”) [4], which can, in the general case, contradict one another. Solution of the inverse problem by the reconstructive computational diagnostics (RCD) aims to identify any specific “sense” which by its very nature is whole (hence can be represented as invariant structure). In a noisy radiographic image of the object we can identify different “semantic structures” by testing the group-theoretical statistical hypotheses with hypothetical local Lie group \( L_\beta \) being the main formalized tool of the group-theoretical filtering [4]. Recognition and identification of local anisotropic images based on local group \( SO(2) \) may serve as an example [4]. This method is easily generalized to tomographic problems.

At studying “dynamic situation”, a specific “semantic structure” appears in the information stream (in a series of images of the evolving object) with its characteristic period \( T \). All the other structures (with other incommensurable periods) will be in disharmony with the set of images with period \( T \). In this situation, more favorable “projections” (i.e., realizations in which other structures are “noises”) appear for the selected structure, and it may be reconstructed with a substantially greater detailing than on the basis of only one “projection”.

From the above, it is easy to understand that the RCD for “dynamic testing” is to be designed as a stroboscopic one, which is consistent with the concept of group-theoretical methods of reconstruction. It is known that stroboscopic imaging is referred to as extraction of a useful signal with distinct characteristics from the signal sequence. Thus, stroboscopic technique includes some prefiltering at the stage of data collection. As indicated in the stroboscopic methodology, it is assumed that in the RCD the information flow from the “dynamic” test object is recorded only at the instants of time when a strobe signal is fed to its data collection subsystem. At all other instants of time, data logging is blocked.

In contrast to classical stroboscopes, the RCD running in a stroboscopic mode will imply that within the time interval between \( t = kT \) and \( t = (k+1)T \), a specific reconstructive inverse problem is solved, and its solution is “displayed” on the recording device (that can be understood literally, for example, a tomogram of a “dynamic object” with a given depth is displayed on the screen).

**Stroboscopic Reconstructive Diagnostics in Phase Time**

As the real rhythms of the open systems are inaccurate, classical stroboscopic methods are...
ineffective for their research. The stroboscopic technique applied to inaccurate periodic processes can be improved with a more flexible approach to the concept of “time”. Along with the conventional “physical” time, which uses the variable \( t \), we introduce phase time \( \tau \), which characterizes a specific phase \( \theta \) of the cyclic process, so that a one-to-one correspondence between \( t \) and \( \tau \) is established, i.e.,

\[
t = t(\tau) \quad \text{and} \quad \tau = \tau(t),
\]

where monotone nondecreasing functions are with \( \tau(T) = 2\pi, t(2\pi) = T, \tau = 2\pi k + \theta \), where \( k \) is the number of the cycle and \( \theta = \tau - 2\pi k \), and for a range of the phase change we have \( 0 \leq \theta \leq 2\pi \). The phase time and phase are equal to \( 2\pi \) by module of congruence, i.e., \( \theta = \tau(\mod 2\pi) \). The data acquisition system is supplied by strobe signals as uniformly as in the traditional stroboscopy, but in the phase time (with period \( \theta \)). In particular, it is easy to carry out stroboscopy so that the strobe signals are giving at instants of \( \tau \) phase time corresponding to fixed \( \theta \) phases (with a period \( \theta = 0 \), which is equivalent to \( \theta = 2\pi \)). For small deflections of \( \Delta\theta \) of \( \theta \) from \( 2\pi \) we can see slow evolution of the test object in the phase time with an angular frequency \( \Delta\omega = \Delta\theta/2\pi \), where the direction of the evolution is determined by \( \Delta\theta \). If \( \Delta\theta = 0 \), the object is “static”.

The starting point to introduce the phase time is an apparent assumption that the proximity of the conditions of the tested system (e.g., heart) in the same phases \( \theta \) is higher than those at instants of physical time \( t_1 \) and \( t_2 \) separated by an integer number of periods \( \bar{m} \), i.e., with \( t_1 - t_2 = mT \) (in other words, at instants of time equal to \( T \) by module of congruence). The arbitrary conditions of the test object from the class determined by equation \( \theta_1 = \theta_2 \) (mod \( 2\pi \)) are closer to each other than the arbitrary conditions from class \( t_1 = t_2 \) (mod \( T \)).

The conventional stroboscopy works with precise periods and exact repetition of object conditions within these periods. Here we mean “inaccurate” or “lean” symmetry of conditions for different realizations corresponding to \( \theta \) phase. The conditions with the same \( \theta \) phase are interconnected in pairs by the equivalence relation and united into equivalence classes \( CL_\theta \), with a system of \( CH_\theta \) features for each of the classes. In this case, the class of \( CL_\theta \), close conditions is defined as the class of comparisons \( \theta_i = \theta_j \) (mod \( 2\pi \)). Any realization can be converted to another one with the same phase by similarity transformation, so that all the features of \( CH_\theta \) are preserved. These transformations form a transitive group \([10] G_\theta \) where the operators are all possible transitions from condition \( k \) to condition \( k' \), where \( k \) and \( k' \) are numbers of realizations \( k = 1, 2, ..., K \).

In the history of science we can find precedents resembling the case of introducing “phase time”. Chizhevsky, who studied the solar-terrestrial relations and the effect of solar activity on a variety of biological and social processes [10], compared different realizations of the 11-yr cycle of solar activity. It is inaccurate. Realizations within the periods of 7 and 17 yrs could be observed. To draw valid conclusions from the experimental data, they should be brought to a unified scale, i.e., to apply phase concept of a cycle. Chizhevsky called this “the method of superposition of epochs” [10].

This “calibration” of data shifts the problem toward ideological basis of “synchronicity” allowing exploration of the evolving objects within the methodology of a structured approach and on the basis of its formal technique. A decisive step is to establish an isomorphism between the cycles. Of course, it can be implemented in many ways keeping the order of phases \( \theta \). It is possible due to monotony of transformations.

**ECG-Controlled Reconstructive Medical Diagnostics**

The transformation laws applied to cardiology cannot be obtained for a great number of cycles because heart is quite a capricious “mechanism”. The symptoms of tachycardia or bradycardia may occur unexpectedly under the impact of various factors. Therefore, to predict the physical time point \( t \) for applying a strobe signal to the data acquisition system according to the law \( t = t(\tau) \), the correction at each cycle is carried out according to the principle “prediction-correction” with regard to the measurement parameters of the electrocardiogram (ECG) results (prima facie the wave positions) during some previous cycles.

This inevitably imposes special requirements to the quality of the recorded ECG. Conventional electrocardiographs have sensitivity level of 20 mm, 10 mm, 5 mm/1 mV, and rather narrow frequency range from 0.05 Hz to
40-100/150 Hz for conventional electrocardiographs and the range from 0.05 Hz to 250 Hz for high resolution electrocardiographs and this enables to accurately determine the values of the amplitude-time parameters of the ECG waves. In Ref [11], it is shown that an accurate description of the ECG waves both by level and by time is provided by a nanoelectrode-based electrocardiograph with the following parameters:

- the input voltage ranges from \( \pm 0.0002 \text{ mV} \) to \( \pm 20 \text{ mV} \) (according to GOST 19867-89, the range is from 0.03 mV to 5 mV);
- the level of the internal peak-to-peak noise ranges from \( +0.1 \mu\text{V} \) to \( -0.1 \mu\text{V} \) (according to GOST 19867-89, the level of noise is less than 20 \( \mu\text{V} \));
- frequency range is \( (0-20)/(0-40)/(0-75)/(0-100)/(0-10,000) \text{ Hz} \);
- the number of channel is 1-3;
- the number of electrode is 3-7;
- ECG transmission to the remote terminal using the accumulation of information on the flash memory installed in the ECG apparatus or via the direct communication channel to PC using the USB port.

The considered electrocardiograph allows measuring the ECG fragments equal to 1 \( \mu\text{V} \), less than 1 \( \mu\text{V} \) or several microvolts in the expanded frequency range from DC to 10,000 Hz with the resolution of 20 nV.

Figure 1 shows the ECG recording of patient P49 with an anterior myocardial.

As can be seen in Figure 1(a), the P-wave is 15 mV and the U-wave is 5 mV. Low amplitude fluctuations of hundreds of nanovolts are shown in Figure 1(b).

Figure 2 shows a graph that represents the heart rate (HR) of patient P49. The graph proves the instability of the period of cardiac cycle which varies from 71 bpm to 74 bpm.

High resolution in ECG measurement in combination with representation of the ECG in the phase time opens up new possibilities for studying the cardiovascular system. This primarily relates to identification and evaluation of the characteristics invariant to transformations from \( G_q \) group.

To put it simply, it is possible to extract and study the characteristics which are common to phase \( \theta \) in each cardiac cycle. This type of problem is effectively solved in the framework of the group-theoretical statistical methods [4-7].

In this case, the initial data available to researchers is cleared of the “semantic noise”, i.e., “noninvariant properties” which are found to be “noises” for certain realizations. This process is similar to the “separation of the layers” in “nonlinear tomosynthesis” [4-6] and is performed within the same conceptual framework. However, extracting “semantic layers”, each of which is characterized by parameter \( \theta \), is considered to be more appropriate in this case.

Figure 1: High resolution ECG recording: (a) Low amplitude P-wave and U-wave (b) Fragment of ECG recording at 103 s.
To sum it up, high resolution in the ECG measurement in combination with ECG representation in the phase time, as well as the development of the group-theoretical statistical approach to the reconstruction of objects, open the way to a comprehensive study of the cardiology problems using mathematical methods.

**Solving the Reconstruction Problems for the Evolving Objects on the Basis of Group-Theoretical Statistical Methods**

The reconstruction problems for the evolving objects, which are solved within the framework of the structural (group-theoretical statistical) approach, fall into two categories:

- “quasi-static” or “purely spatial” problems, each of which is solved for a specific moment of phase time $\theta$, i.e., the dependence of the solution on time is parametric;
- nontrivial “dynamic” problems with spatio-temporal invariants of an evolving object in their solutions.

**“Quasi-Static” Problems**

This type of problem can be successfully solved using the “mathematical technique” of the group-theoretical statistical methods developed to solve the problems in nondestructive testing [4-6]. This refers to both the problems of tomography [6] and image reconstruction [4]. In a typical situation, a set of statistical problems are solved for the moments of time $\tau = 2\pi k + \theta$ with $k = 1, 2, ..., K$ at a fixed phase $\theta$. These problems are solved separately and parameter $\theta$ for the relationship of the conditions is not considered. Nevertheless, these solutions give much information about the object in condition with parameter $\theta$, in particular, the possibility of making a comparative visual analysis of solutions, detecting invariants and deviations from them in separate solutions, as well as tracking the impact of various factors on such deviations. This makes the basis for development of new algorithms and software for postprocessing the solutions.

**“Dynamic” Problems**

Primal internal instability is the “motor” for self-development of any open system, in particular,
an organism. An important aspect of this development is the dynamic stability, that is, preservation by the system certain dynamical structural invariants on a particular “significant” stage of its evolution. This fact allows us to treat the dynamic stability as a self-identical object. In the “dynamic” problems, relationship of $K$ conditions of the object by parameter $\theta$ is taken into account from the outset, and the set $k = 1, 2, \ldots, K$ is considered as a structural unity.

The investigation of the internal dynamic instabilities of the cardiovascular system is crucial for diagnostics of pathologies associated with the cardiovascular system. The most evident instability associated with heart rhythm disorder is “weak”, and it is not an indicator of a certain disease. Rather, it is an effective means of adapting to a variety of effects on the body (both external and internal). The early diagnostics is essential for pathological instabilities which have a tendency to the rapid (“nonlinear”) development.

The establishment of the “normal” condition for a test object and the detection of deviations from the normal condition under the impact of various factors are typical “dynamic” problems. In fact, dynamic problems are the problems of the mathematical description of homeostasis (in particular, maintaining the stability of the body’s essential variables) on the basis of group-theoretical statistical methods.

From the standpoint of biology and medicine, generally accepted standards of health can hardly be defined. It would be natural to consider different “nорм” for different groups of people, but the scope of this research refers to the development of the methods to formalize the “norm” as the structure described by its structural invariants.

When the state of an organism is normal and “quiet”, the invariants can include such characteristics as heart rate, systolic and diastolic blood pressure, systolic and minute volume of blood, etc. If a set of organism conditions is extended by the stress influence, a set of invariant characteristics will be narrow. The assessment of relaxation time of the body after the impact is also an important category of “dynamic” problems of diagnostics.

A simple example of the stress influence is a stressful physical activity, when the heart rate (obviously noninvariant value) rises to 220 beats per minute or more. Comparing a set of the heart roentgenograms at a certain precisely fixed phase with parameter $\theta$ within the period, when the heart rate drops to normal, is valuable for the development of diagnostic methods. In the case of critical body burden (for a given body), some small changes in the “invariant” characteristics can be identified.

As a part of the group-theoretical statistical approach [4,5], the problem of discriminating norms and anomalies in the test object is reduced to the problem of discriminating statistically significant symmetry and asymmetry in the information image of the test object (set of X-ray projections). The solution is based on testing the group-theoretical statistical hypothesis on the invariance of the information image relative to a hypothetical local Lie group $L_0$ [4]. The “maintenance” of the invariant characteristics of the organism within the “norm” is a zero hypothesis. A statistically significant deviation (corresponding to a certain anomaly) from the invariant is the alternative hypothesis. Decreasing the symmetry of the structure (in particular, the structure described by group $G_0$) caused by this characteristic is evaluated on the basis of Curie’s dissymmetrization [5], and the statistical “measures of dissimilarity” are elaborated for the quantitative estimation of the symmetry reduction [4].

In the same way as the result of the structural-oriented NDT can be represented in the form of a defect map [5] plotted on the basis of the group-theoretical approaches, a “pathology map” based on the same principles can be introduced in medical diagnostics. Emphasize that the map gives a “synchronic” description of the body. The map should be plotted not in a three-dimensional space, but in some other configuration spaces reflecting disorders of the most important (in terms of the established “norm”) structure–function relationships of the body and representing thus a kind of a “structure–function portrait” of a patient.

**Result and Conclusions**

Positive experience in solving some of the problems in nondestructive testing using group-theoretical statistical reconstructive methods has allowed us to develop similar methods for solving the problems of reconstruction of invariant characteristics of the evolving objects, primarily to solve diagnostic problems in cardiology.

The most significant findings are as follows:

1. A structural approach to the evolving objects as an open system is developed. The
The essence of the inverse reconstructive problem in terms of the structural approach is investigated. The features of the inverse problems for developing open systems are considered in details. Application of the stroboscopic methods of data recording in computer diagnostics for reconstruction of the evolving objects is proposed and justified.

2. The problem of the inaccurate rhythms calibration of the evolving objects is solved and the concept of phase time is introduced. The methods of stroboscopic reconstructive diagnostics in phase time are developed.

3. The ECG-controlled systems for reconstructive medical diagnostics are proposed. We have explored the possibilities of their hardware implementation on the basis of the electrocardiograph, which enables measuring the ECG fragments equal to one microvolt, less than one microvolt or several microvolts in the expanded frequency range from DC to 10,000 Hz with the resolution of 20 nV.

4. The major types of “static” and “dynamic” problems in the reconstruction of the invariant characteristics of the evolving objects are considered. The issues of formalizing the “norm” of testing and identifying the anomalies such as the background symmetry reduction under the impact of external factors are discussed within the framework of the group-theoretical statistical methods.

The potential scope of applications of the mathematical methods of reconstruction of the evolving objects is considered to be broad. These methods can be applied not only in medical diagnostics but also in geophysics, astrophysics, optimizing management in the “man–machine” system, etc. The major theoretical issues concerning the development of the ECG-controlled X-ray systems for medical diagnostics have been solved. The obtained results can be successfully applied in a vast majority of areas.

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